S je sumacioni vektor, ta da je $s^t s$ = ?

$$s^t s = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = 1*1+1*1+1*1+1*1=4$$

$s^t R s$ =

$$s^t R s = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 2 & 0 & 3 \\ 1 & 2 & 4 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

$$s^t R s = \begin{bmatrix} 2 & 7 & 9 & 15 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = 33$$

Projektor

A razapinje njeki prostor

$P = PP$

$P_A = A(A^tA)^{-1}A^t$

$$\begin{aligned} P_A P_A &= A(A^tA)^{-1}(A^t A)(A^tA)^{-1}A^t \\ &= A(A^tA)^{-1}IA^t \\ &= A(A^tA)^{-1}A^t \\ &= P_A \end{aligned}$$

$P_A x = x'$ slika vektora, kooridinate vektora u prostoru kolona matrice A

$x - P_A x = (I - P_A)\, x = x''$ deo vektora x koji štrči iz prostora

$$\begin{aligned} P_{A'} &= (I - P_A) \\ P_{A'} P_{A'} &= (I - P_A)(I - P_A) \\ &= I\,I - IP_A - P_A I + P_A P_A \\ &= I - P_A - P_A + P_A \\ &= I - P_A \end{aligned}$$

Izvodi

Projektor od centroidnog (sumacionog) vektora

$P_s \quad = s\,(s^t s)^{-1} s^t$

$\quad = s\, n^{-1}\, s^t$

$\quad =$

$$s s^t n^{-1} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix} 4^{-1} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} 4^{-1}$$

$$s s^t n^{-1} = \begin{bmatrix} 1/4 & 1/4 & 1/4 & 1/4 \\ 1/4 & 1/4 & 1/4 & 1/4 \\ 1/4 & 1/4 & 1/4 & 1/4 \\ 1/4 & 1/4 & 1/4 & 1/4 \end{bmatrix}$$

$P_s x =$

$$s s^t n^{-1} x = \begin{bmatrix} 1/4 & 1/4 & 1/4 & 1/4 \\ 1/4 & 1/4 & 1/4 & 1/4 \\ 1/4 & 1/4 & 1/4 & 1/4 \\ 1/4 & 1/4 & 1/4 & 1/4 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 4 \\ 4 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ 3 \\ 3 \end{bmatrix}$$

$(I - P_s) = I - s\,(s^t s)^{-1} s^t$

$x' = (I - P_s)x = x - P_s x$

$$\begin{bmatrix} 1 \\ 3 \\ 4 \\ 4 \end{bmatrix} - \begin{bmatrix} 3 \\ 3 \\ 3 \\ 3 \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \\ 1 \\ 1 \end{bmatrix}$$

$x'^t x' n^{-1} = \begin{bmatrix} -2 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} -2 \\ 0 \\ 1 \\ 1 \end{bmatrix} 4^{-1} = 6/4 = 1.5 = \text{Varijansa}$

Regresija

Zx = k' = k – e

f(e) = najmanji kvadrat = $e^t e$

e = k - Zx

$f(x) = e^t e = (k - Zx)^t (k - Zx)$
$= (k^t - x^t Z^t)(k - Zx)$
$= k^t k - k^t Zx - x^t Z^t k + x^t Z^t Zx$
$= k^t k - 2\,x^t Z^t k + x^t Z^t Zx$ = minimum (najmanji kvadrat)

$$f(x)' = \frac{\partial f(x)}{\partial x} = 0 - 2Z^t k + 2Z^t Zx$$

$$-2Z^t k + 2Z^t Zx = 0$$
$$-Z^t k + Z^t Zx = 0$$
$$Z^t Zx = Z^t k$$
$$(Z^t Z)^{-1} Z^t Zx = (Z^t Z)^{-1} Z^t k$$
$$x = (Z^t Z)^{-1} Z^t k$$

predividjeni skorovi

$$k' = Zx = Z(Z^t Z)^{-1} Z^t k = P_z k$$
$$e = k - k' = k - Zx = k - Z(Z^t Z)^{-1} Z^t k = k - P_z k = (I - P_z)k$$

Domaci

Treba napraviti program za racunanje multiple regresije, u matricnom obliku. U SPSS-u napravite standradizovane 3 koolone sa 5 isptiaanika i onda te vrednosti prenesetu u R i proverite vas racun i SPSS.

Svojstvene vrednosti i svojstveni vektori – eigenvalues & eigenvectors

A – kvadratna matrica

A x = λ x

λ skalara, svojstvena vrednost

x je vektor svojstveni vektor

A x - λ x = 0

(A - Iλ) x = 0

| A - Iλ | = 0

dimenzije matrice A je m

$$\Lambda = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}$$

$$X = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}$$

$$AX = X\Lambda$$

$$AXX^{-1} = A = X\Lambda X^{-1}$$

$$X^{-1}AX = X^{-1}X\Lambda = \Lambda$$

ako je matrica A simetricna, onda je X ortogonalno $X^tX = I$

$$AX = X\Lambda$$

$$AXX^t = A = X\Lambda X^t$$

$$X^t AX = X^t X\Lambda = \Lambda$$

Lagrange multipliers

Principal components

Z – standardizovane varijable, u standoardnoj normalnoj formi

Z^tZ = R matrica inerkorelacija

$Zx = k$, k^tk = varijansa = zelimo maximum, pod uslovom da je $x^tx = 1$

$f(x) = k^tk = (Zx)^t(Zx) = x^tZ^tZx = x^tRx$ = maximum

$g(x) = x^tx - 1$

$L(x, \lambda) = f(x) - \lambda g(x)$

$= x^tRx - \lambda(x^tIx - 1)$

$$\frac{\partial L}{\partial x} = 2Rx - \lambda 2Ix = 0$$

$$\frac{\partial L}{\partial \lambda} = x^t x - 1 = 0$$

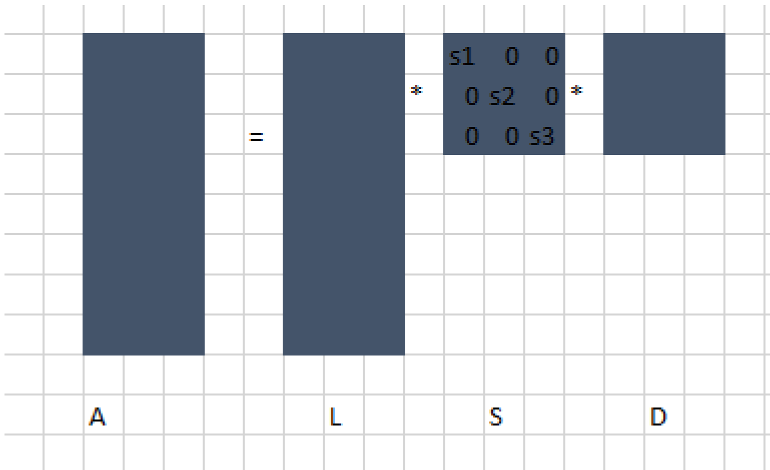
$$(R - \lambda I)x = 0$$

$$|R - \lambda I| = 0$$

λ – varijanse glavnih komponenti

x – koeficijenti za dovijanje glavnih komponenti k = Zx

Singular Value Decomposition



$A = LSD^t$

S = diag(S) – matrica sa singularnim vrednostima

$L^tL = I$

$D^tD = DD^t = I$

D je matrica svojstvenih vektora

$A^tA = (LSD^t)^t (LSD^t)$

$= (D^t)^t S^t L^t LSD^t$

$= DSL^tLSD^t$

$= D S^2 D^t$

iz svojstvenih vrednosti $AXX^t = A = X \Lambda X^t$

sto znaci da je D = X, a $\Lambda = S^2$

Generalized inverse

Je ustvari SVD

$A = LSD^t$

tada je generalizovani invez matrice $\mathbf{A^* = LS^{-1}D^t}$

Optimization algorithms

Newton's method

Quasi-Newton method

Gradient descent

R

Domaci

https://machinelearningmastery.com/logistic-regression-tutorial-for-machine-learning/