

S je sumacioni vektor, ta da je $s^t s = ?$

$$s^t s = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = 1*1 + 1*1 + 1*1 + 1*1 = 4$$

$s^t R s =$

$$s^t R s = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 2 & 0 & 3 \\ 1 & 2 & 4 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

$$s^t R s = \begin{bmatrix} 2 & 7 & 9 & 15 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = 33$$

Projektor

A razapinje neki prostor

$$P = P^2$$

$$P_A = A(A^t A)^{-1} A^t$$

$$P_A P_A = A(A^t A)^{-1} (A^t A) (A^t A)^{-1} A^t$$

$$= A(A^t A)^{-1} I A^t$$

$$= A(A^t A)^{-1} A^t$$

$$= P_A$$

$P_A x = x'$ slika vektora, koordinate vektora u prostoru kolona matrice A

$x - P_A x = (I - P_A) x = x''$ deo vektora x koji štrči iz prostora

$$P_{A'} = (I - P_A)$$

$$P_{A'} P_{A'} = (I - P_A)(I - P_A)$$

$$= I I - I P_A - P_A I + P_A P_A$$

$$= I - P_A - P_A + P_A$$

$$= I - P_A$$

Izvodi

Projektor od centroidnog (sumacionog) vektora

$$\begin{aligned} P_s &= s (s^t s)^{-1} s^t \\ &= s n^{-1} s^t \\ &= \end{aligned}$$

$$s s^t n^{-1} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} [1 \quad 1 \quad 1 \quad 1] 4^{-1} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} 4^{-1}$$

$$s s^t n^{-1} = \begin{bmatrix} 1/4 & 1/4 & 1/4 & 1/4 \\ 1/4 & 1/4 & 1/4 & 1/4 \\ 1/4 & 1/4 & 1/4 & 1/4 \\ 1/4 & 1/4 & 1/4 & 1/4 \end{bmatrix}$$

$P_s x =$

$$s s^t n^{-1} x = \begin{bmatrix} 1/4 & 1/4 & 1/4 & 1/4 \\ 1/4 & 1/4 & 1/4 & 1/4 \\ 1/4 & 1/4 & 1/4 & 1/4 \\ 1/4 & 1/4 & 1/4 & 1/4 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 4 \\ 4 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ 3 \\ 3 \end{bmatrix}$$

$(I - P_s) = I - s (s^t s)^{-1} s^t$

$x' = (I - P_s)x = x - P_s x$

$$\begin{bmatrix} 1 \\ 3 \\ 4 \\ 4 \end{bmatrix} - \begin{bmatrix} 3 \\ 3 \\ 3 \\ 3 \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \\ 1 \\ 1 \end{bmatrix}$$

$$x^t x^t n^{-1} = \begin{bmatrix} -2 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} -2 \\ 0 \\ 1 \\ 1 \end{bmatrix} 4^{-1} = 6/4 = 1.5 = \text{Varijansa}$$

Regresija

$$Zx = k' = k - e$$

$$f(e) = \text{najmanji kvadrat} = e'e$$

$$e = k - Zx$$

$$\begin{aligned} f(x) &= e'e = (k - Zx)'(k - Zx) \\ &= (k' - x'Z') (k - Zx) \\ &= k'k - k'Zx - x'Z'k + x'Z'Zx \\ &= k'k - 2 x'Z'k + x'Z'Zx = \text{minimum (najmanji kvadrat)} \end{aligned}$$

$$f(x)' = \frac{\partial f(x)}{\partial x} = 0 - 2 Z^t k + 2 Z^t Zx$$

$$-2 Z^t k + 2 Z^t Zx = 0$$

$$-Z^t k + Z^t Zx = 0$$

$$Z^t Zx = Z^t k$$

$$(Z^t Z)^{-1} Z^t Zx = (Z^t Z)^{-1} Z^t k$$

$$x = (Z^t Z)^{-1} Z^t k$$

previdjeni skorovi

$$k' = Zx = Z (Z^t Z)^{-1} Z^t k = P_z k$$

$$e = k - k' = k - Zx = k - Z (Z^t Z)^{-1} Z^t k = k - P_z k = (I - P_z) k$$

Domaci

Treba napraviti program za racunanje multiple regresije, u matricnom obliku. U SPSS-u napravite standradizovane 3 koolone sa 5 isptiaanika i onda te vrednosti prenesetu u R i proverite vas racun i SPSS.

Svojstvene vrednosti i svojstveni vektori – eigenvalues & eigenvectors

A – kvadratna matrica

$$A x = \lambda x$$

λ skalara, svojstvena vrednost

x je vektor svojstveni vektor

$$A x - \lambda x = 0$$

$$(A - I\lambda) x = 0$$

$$|A - I\lambda| = 0$$

dimenzije matrice A je m

$$\Lambda = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}$$

$$X = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}$$

$$AX = X\Lambda$$

$$AXX^{-1} = A = X\Lambda X^{-1}$$

$$X^{-1}AX = X^{-1}X\Lambda = \Lambda$$

ako je matrica A simetricna, onda je X ortogonalno $X^tX = I$

$$AX = X\Lambda$$

$$AXX^t = A = X\Lambda X^t$$

$$X^t AX = X^t X\Lambda = \Lambda$$